Project #2 - Beam Deflection

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12/15/25

Part 1: Bending of Steel and Aluminum Beams

Givens:

- Beams' cross section: 1.5875 mm x 12.7 mm
- Steel's Elastic modulus (E) = 200 GPa
- Aluminum's Elastic modulus (E) = 70 GPa

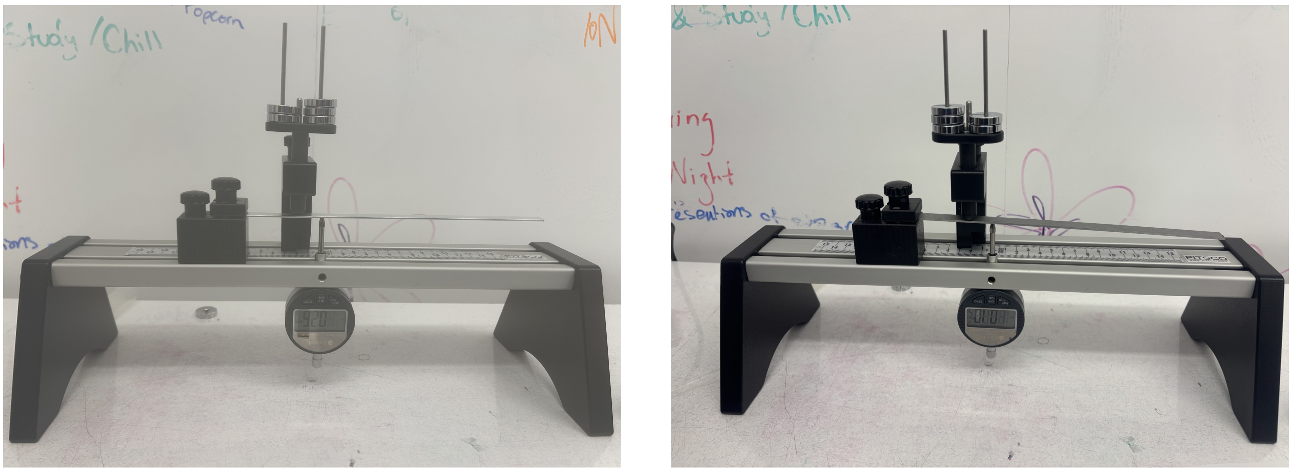
**Note that throughout this report we choose to ignore the distributed weight of the beam to simplify the calculations (as instructed).

Experimental Calculations

We performed testing for this section on both steel and aluminum. The two support conditions were simply supported and cantilevered. The load for both conditions and both materials was 100g (1N). The following data were recorded:

Cantilevered:

- deflection of Steel: 0.1 mm
- deflection of Aluminum: 0.26 mm



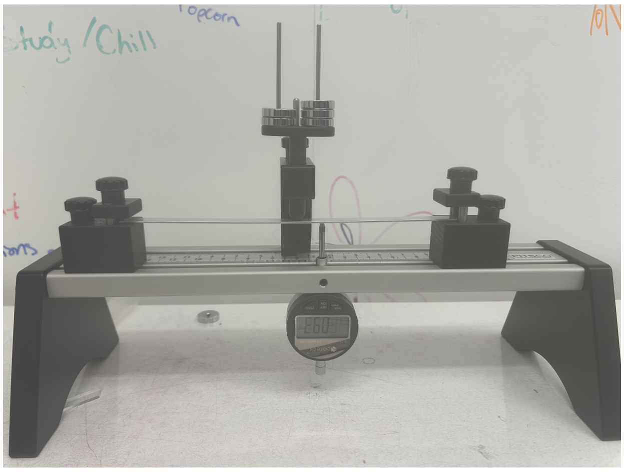
(a) Cantilevered: Aluminum Under 100g Load

(b) Cantilevered: Steel Under 100g Load

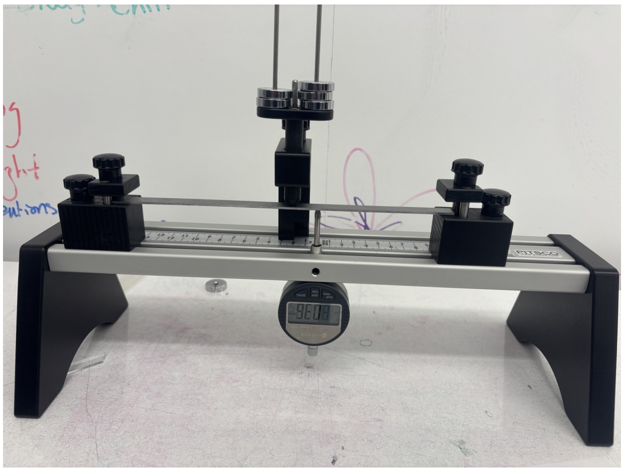
Figure 1: Experimental setup for aluminum and steel cantilever beams.

Simply Supported

- Deflection of aluminum: 0.93 mm
- Deflection of steel: 0.34 mm



(a) Simply Supported: Aluminum Under 100g Load



(b) Simply Supported: Steel Under 100g Load

Figure 2: Comparison of simply supported experimental setups

Theoretical Calculations

Support Condition: Cantilevered

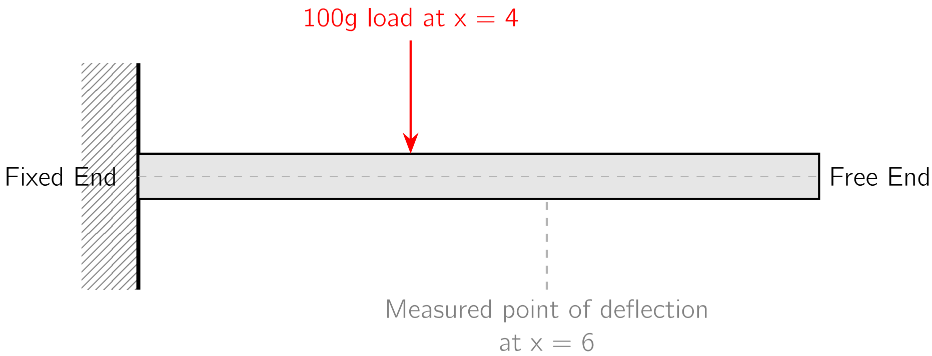
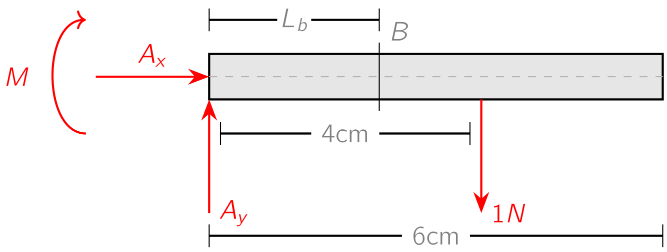


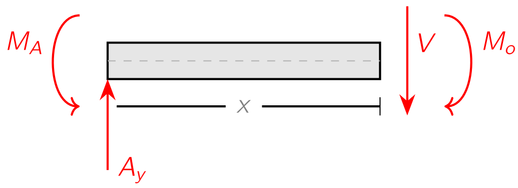
Figure 3: A cantilever beam showing the applied weight force



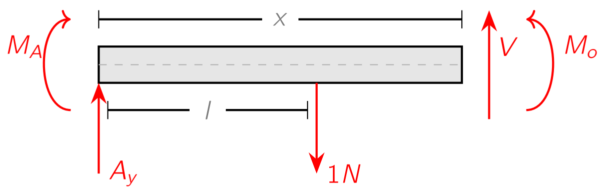
(a) Free-Body Diagram of the Cantilevered Beam

Support Reactions:

- $\sum F_x = \boxed{A_x = 0}$
- $\sum F_y = 0 = A_y - 1\text{N}$
  $\boxed{A_y = 1\text{N}}$
- $\sum M_A = 0 = M - 1(0.04\text{m})$
  $\boxed{M = 0.04\text{Nm}}$



(a) Cut Section $0 < x < 4$



(b) Cut Section $4 < x < 6$

Internal Forces Calculations:

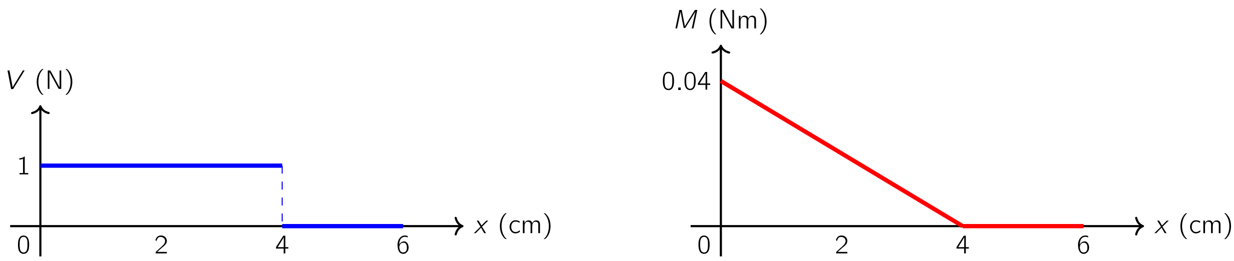
For $0 < x < 4$:

- $\sum F_y = 0 = A_y - V, \boxed{V = 1\text{N}}$
- $\sum M_{\text{cut}} = -x + M_A = -x + 0.04$
- $M(0.04) = 0.04 - 0.04, 0 = 0 + C_1, C_1 = 0$

For $4 < x < 6$:

- $\sum F_y = 0 = A_y - 1\text{N} + V, \boxed{V = 0\text{N}}$
- $\sum M_{\text{cut}} = 0 + C_1$

Figure 5: Shear and Moment Calculations



(a) Shear Force Diagram $V(x)$ (b) Bending Moment Diagram $M(x)$

Figure 6: Internal force diagrams for the Cantilevered beam

Maximum Bending Stress:

- Using the Flexure formula and $M_{max} = 0.04$: $\sigma = \frac{M\cdot c}{I} \rightarrow \sigma_{max} = \frac{0.04\cdot(\frac{1.5875\text{mm}}{2})}{\frac{1}{12}(12.7)(1.5875)^3} = \boxed{7.50 \text{ MPa}}$

Elastic Curve:

- $0 < x < 4$
  - $EIv_1'' = M(x) = x - 0.04$
  - $EIv_1' = \frac{x^2}{2} - 0.04x + C_1$
  - $EIv_1 = \frac{x^3}{6} - \frac{0.04x^2}{2} + C_1x + C_2$
- $4 < x < 6$
  - $EIv_2'' = M(x) = 0$
  - $EIv_2' = C_3$
  - $EIv_2 = C_3x + C_4$

Boundary Condition @ $(x = 0\text{cm}), (v_1 = 0), (\theta_1 = 0)$:

- $EI(0) = \frac{0^3}{6} - \frac{0.04}{2}(0)^2 + C_1(0) + C_2 \rightarrow \boxed{C_2 = 0}$
- $EI(0) = \frac{0^2}{2} - 0.04(0) + C_1 \rightarrow \boxed{C_1 = 0}$

Continuity Condition @$(x = 4\text{cm}), (v_2 = v_1), (\theta_2 = \theta_1)$:

- $EIv_1 = EIv_2 \rightarrow \frac{0.04^3}{6} - 0.02(0.04)^2 = C_3(0.04) + C_4 \rightarrow \boxed{C_4 = 1.067\text{e}{-5}}$
- $EIv_1' = EIv_2' \rightarrow \frac{0.04^2}{2} - 0.04(0.04) = C_3 \rightarrow \boxed{C_3 = -0.0008}$

Elastic Curve Final:

$0 < x < 4$

- $EIv_1 = \frac{x^3}{6} - 0.02x^2$

$4 < x < 6$

- $EIv_2 = -0.0008x + 1.067\text{e}{-5}$

Deflection: We know the moment of inertia to be: $I = \frac{bh^3}{12} = \frac{1}{12}(0.0127)(0.0015875)^3 = 4.23\text{e}{-12}\text{m}^4$

- Steel: $v_2(200e9)(4.23e{-}12) = -0.0012(0.06) + 9.07e{-}5 \rightarrow v_2 = 1.4e{-}5\text{m} = \boxed{v_2 = 0.044 \text{ mm}}$
- Aluminum: $v_2(70e9)(6.67e{-}12) = -0.0012(0.06) + 9.07e{-}5 \rightarrow \boxed{v_2 = 0.126 \text{ mm}}$

Check Against Theory:

- Steel: $v = \frac{-PL^3}{48EI}(6x - L) \rightarrow v = [\frac{-(1\text{N})(0.08)^2}{48(200e9)(4.23e{-}12)}][6(0.06) - 0.08] \rightarrow \boxed{v_2 = 0.044 \text{ mm}}$
- Aluminum: $v = \frac{-PL^3}{48EI}(6x - L) \rightarrow v = [\frac{-(1\text{N})(0.08)^2}{48(70e9)(4.23e{-}12)}][6(0.06) - 0.08] \rightarrow \boxed{v_2 = 0.126 \text{ mm}}$

This checks out!!

Comparison

*See attached Excel File

Support Condition: Simply Supported

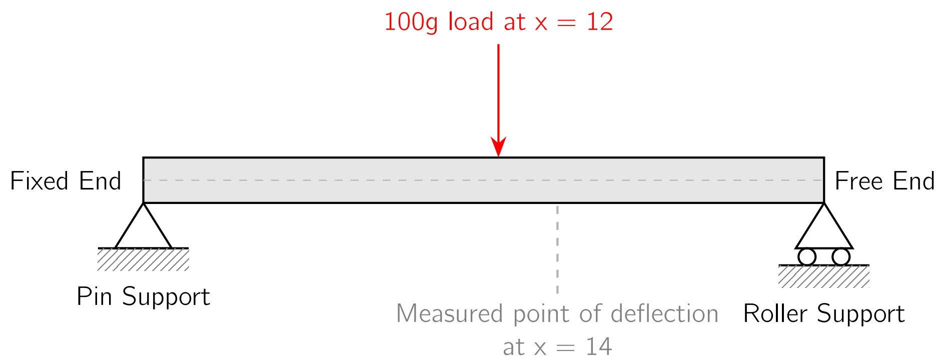
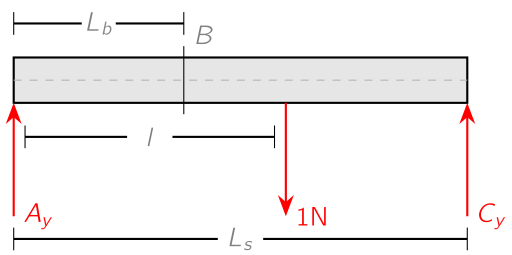


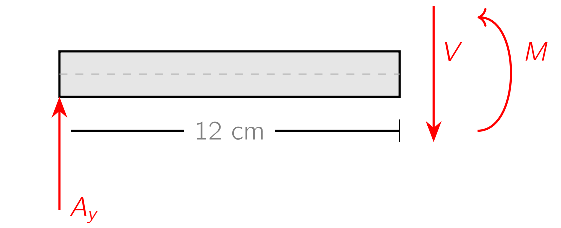
Figure 9: A simply supported beam showing the applied load



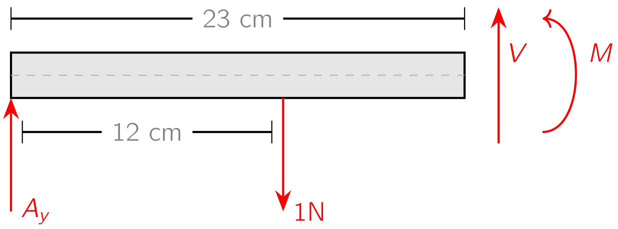
(a) Loading Condition for Simply Supported Beam

Support Reactions:

- $\sum F_x = 0$
- $\sum F_y = 0 = C_y + A_y - 1\text{N}$
- $\sum M_A = 0 = -1(0.12\text{m}) + C_y(0.23\text{m})$
  $\boxed{C_y = 0.522\text{N}}, \boxed{A_y = 0.478\text{N}}$



(a) Cut section $0 < x < 12$



(b) Cut section $12 < x < 23$

Internal Forces Calculations:

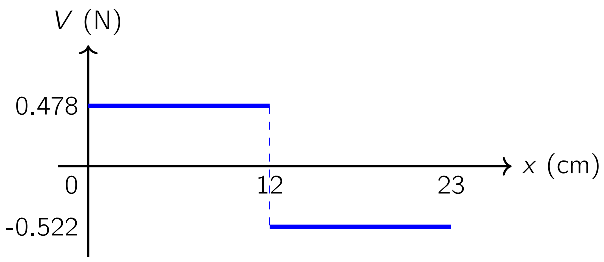
For $0 < x < 12$:

- $\sum F_y = 0 = A_y - V \rightarrow A_y = \boxed{V = 0.478 \text{ N}}$
- $\sum M_{cut} = 0 = -A_y x - M_1 \rightarrow M_1 = A_y x$
- $\boxed{M_1 = 0.478x \text{ Nm}}$

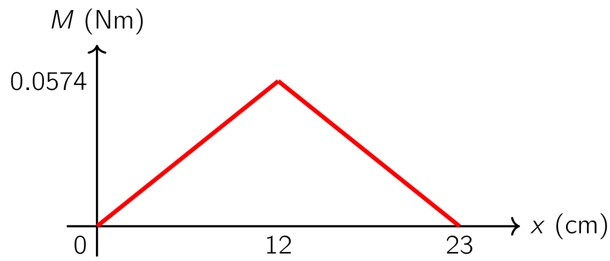
For $12 < x < 23$:

- $\sum F_y = 0 = A_y - 1\text{N} - V, \boxed{V = -0.522 \text{ N}}$
- $\sum M_{cut} = 0 = -A_y x + 1(x - 0.12) - M_2$
- $\boxed{M_2 = -0.478x + 0.12 \text{ Nm}}$

Figure 11: Shear and Moment Calculations for Simply Supported Beam



(a) Shear Force Diagram $V(x)$



(b) Bending Moment Diagram $M(x)$

Figure 12: Internal force diagrams for Simply Supported Beam

Maximum Bending Stress:

- Using the Flexure formula and $M_{max} = 0.12$: $\sigma = \frac{M \cdot c}{I} \rightarrow \sigma_{max} = \frac{0.0574 \cdot (\frac{1.5875mm}{2})}{\frac{1}{12}(12.7)(1.5875)^3} = \boxed{10.76 \text{ MPa}}$

Elastic Curve:

- $0 < x < 12$
  - $EIv_1'' = M(x) = 0.4783x$
  - $EIv_1' = \frac{0.4783}{2}x^2 + C_1$
  - $EIv_1 = \frac{0.4783}{6}x^3 + C_1 x + C_2$
- $12 < x < 23$
  - $EIv_2'' = M(x) = 0.5217x - 0.0574$
  - $EIv_2' = 0.5217\frac{1}{2}x^2 - 0.0574x + C_3$
  - $EIv_2 = 0.5217\frac{x^3}{6} - 0.0574\frac{x^2}{2} + C_3 x + C_4$

Boundary Conditions:

- @ $(x = 0\text{cm}), (v_1 = 0), (\theta_1 = 0)$
  - $EI(0) = \frac{0.4783}{6}(0)^3 + C_1(0) + C_2 \rightarrow \boxed{C_2 = 0}$
  - $EI(0) = \frac{0.4783}{2}(0)^2 + C_1 \rightarrow \boxed{C_1 = 0}$
- @ $(x = 0.23\text{cm}), (v_2 = 0), (\theta_2 = 0 = v_2')$
  - $EI(0) = 0.5217(\frac{0.23^3}{6}) - 0.0574\frac{0.23^2}{2} + C_3(0.23) + C_4 \rightarrow 4.603e{-4} = 0.23C_3 + C_4$
  - $EI(0) = \frac{0.5217}{2}(0.23)^2 - 0.0574(0.23) + C_3 \rightarrow \boxed{C_3 = -5.97e{-4}}, \boxed{C_4 = 5.976e{-4}}$

Continuity Condition @$(x = 12\text{cm}), (v_2 = v_1), (\theta_2 = \theta_1)$:

- $0.4783\frac{0.12^3}{6} + C_1(0.12) = 0.5217\frac{0.12^3}{6} - 0.0574\frac{0.12^2}{2} + C_3(0.12) + C_4$

Elastic Curve Final:

$0 < x < 12$

- $EIv_1 = \frac{0.4783}{6}x^3$

$12 < x < 23$

- $EIv_2 = \frac{0.5217}{6}x^3 - \frac{0.0574}{2}x^2 - 0.000597x + 0.0005976$

Deflection: We know the moment of inertia to be: $I = \frac{bh^3}{12} = \frac{1}{12}(0.0127)(0.0015875)^3 = 4.23\text{e}{-12}\text{m}^4$

- Steel: $v_2(200\text{e}9)(4.23\text{e}{-12}) = 1.9\text{e}{-4} \rightarrow \boxed{v_2 = 0.225 \text{ mm}}$
- Aluminum: $v_2(70\text{e}9)(4.23\text{e}{-12}) = 1.9\text{e}{-4} \rightarrow \boxed{v_2 = 0.76 \text{ mm}}$

Check Against Theory:

- Steel: $v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2) \rightarrow v = [\frac{-1(0.12)(0.09)}{6(200\text{e}9)(4.23\text{e}{-12})(0.23)}][0.23^2 - 0.12^2 - 0.09^2] \rightarrow \boxed{v_2 = 0.28 \text{ mm}}$
- Aluminum: $v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2) \rightarrow v = [\frac{-1(0.12)(0.09)}{6(70\text{e}9)(4.23\text{e}{-12})(0.23)}][0.23^2 - 0.12^2 - 0.09^2] \rightarrow \boxed{v_2 = 0.803 \text{ mm}}$

These are fairly close!!

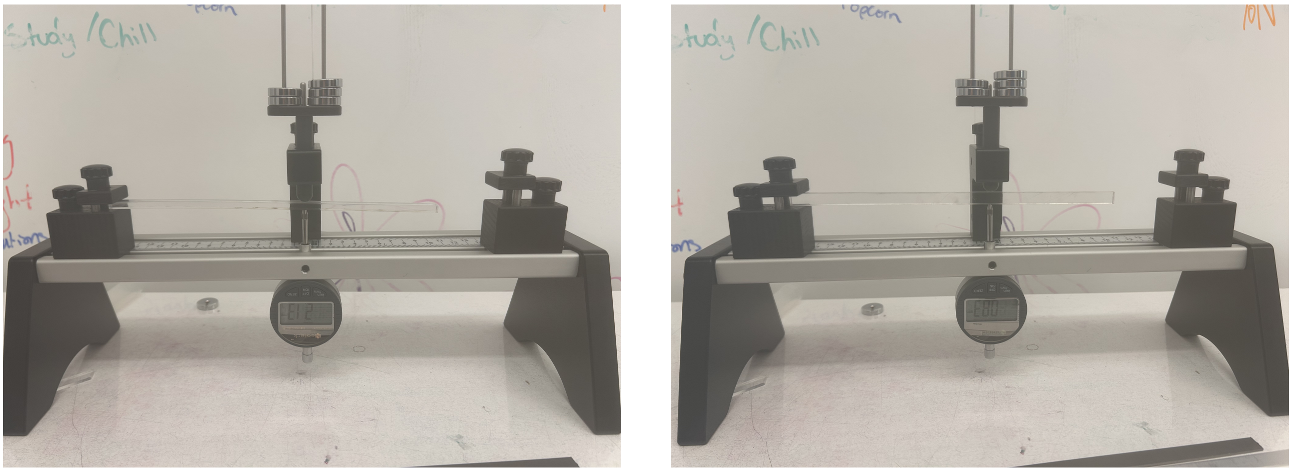
Comparison

*See attached Excel File

Part 2: Bending of Acrylic Rod in two orientations

Experimental Calculations

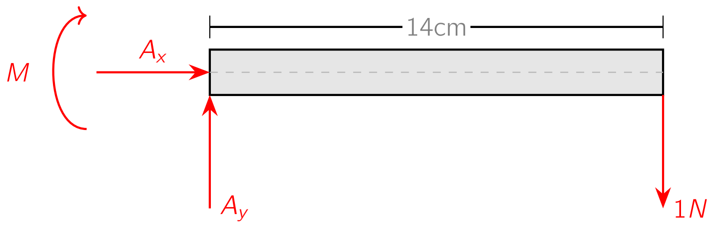
- Deflection of the first orientation (cross section 6.35 mm): 2.13mm
- Deflection of the second orientation (cross section 10.0 mm): 0.83mm



(a) Condition 1 (Lower I) (b) Condition 2 (Higher I)

Figure 15: Experimental comparison of acrylic rod orientations.

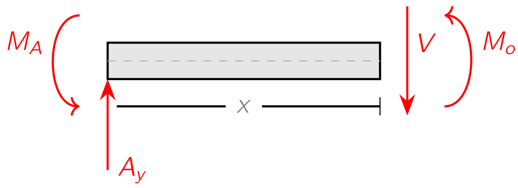
Theoretical Calculations



(a) Free-Body Diagram of the Cantilevered Beam

Support Reactions:

- $\sum F_x = \boxed{A_x = 0}$
- $\sum F_y = 0 = A_y - 1 \text{ N}$ $\boxed{A_y = 1\text{N}}$
- $\sum M_A = 0 = M - 0.14(1)$ $\boxed{M = 0.14 \text{ Nm}}$



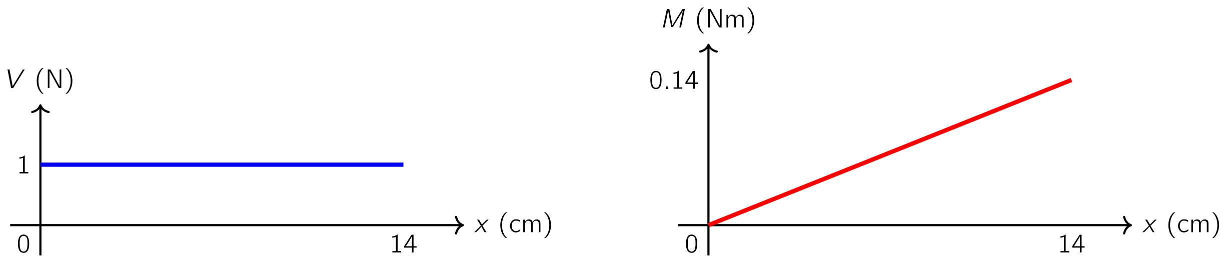
(a) Cut Section $0 < x < 14$

Internal Forces Calculations:

For $0 < x < 14$:

- $\sum F_y = 0 = A_y - 1\text{N} + V$, $\boxed{V = 0\text{N}}$
- $\sum M_{\text{cut}} = 0 + C_1$

Figure 17: Shear and Moment Calculations



(a) Shear Force Diagram $V(x)$ (b) Bending Moment Diagram $M(x)$

Figure 18: Internal force diagrams for the beam calculated in Part 1.

Elastic Curve:

- $EIv_1'' = M(x) = x$
- $EIv_1' = \frac{x^2}{2} + C_1$
- $EIv_1 = \frac{x^3}{6} + C_1 x + C_2$

Boundary Condition @ $(x = 0\text{cm}), (v = 0), (\theta = 0)$:

- $EI(0) = \frac{0^3}{6} + C_1(0) + C_2 \rightarrow \boxed{C_2 = 0}$
- $EI(0) = \frac{0^2}{2} + C_1 \rightarrow \boxed{C_1 = 0}$

Elastic Curve: $v = \frac{1}{EI}\frac{x^3}{6}$, $E = 3$ GPa

Vertical Beam:

- $I = \frac{1}{12}(6.35\text{mm})(10\text{mm})^3 = 5.29\text{e}{-10}\text{mm}^4$
- $v = \frac{1}{(3\text{e}4)(5.29\text{e}-10)}\frac{0.14^3}{3} = \boxed{0.576 \text{ mm}}$
- $\sigma = \frac{Mc}{I} = \frac{(0.14)(0.005)}{(5.29\text{e}-10)} = \boxed{1.32 \text{ MPa}}$

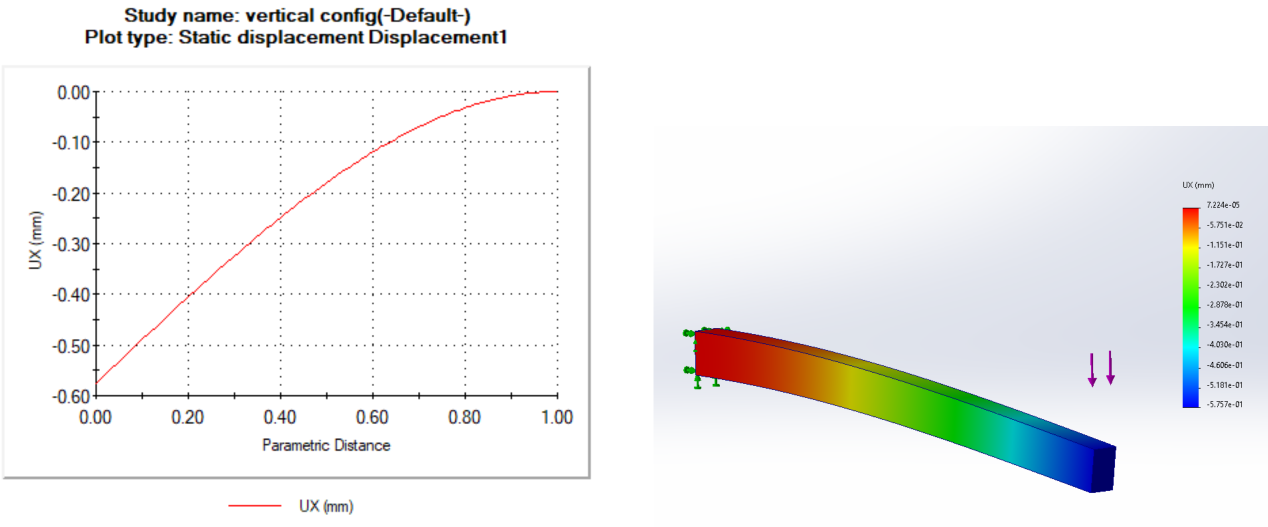
Horizontal Beam:

- $I = \frac{1}{12}(10\text{mm})(6.35\text{mm})^3 = 2.13\text{e}{-10}\text{mm}^4$
- $v = \frac{1}{(3\text{e}4)(2.134\text{e}-10)}\frac{0.14^3}{3} = \boxed{1.4288 \text{ mm}}$
- $\sigma = \frac{Mc}{I} = \frac{(0.14)(0.003175)}{(2.134\text{e}-10)} = \boxed{2.08 \text{ MPa}}$

FEA

Vertical configuration:

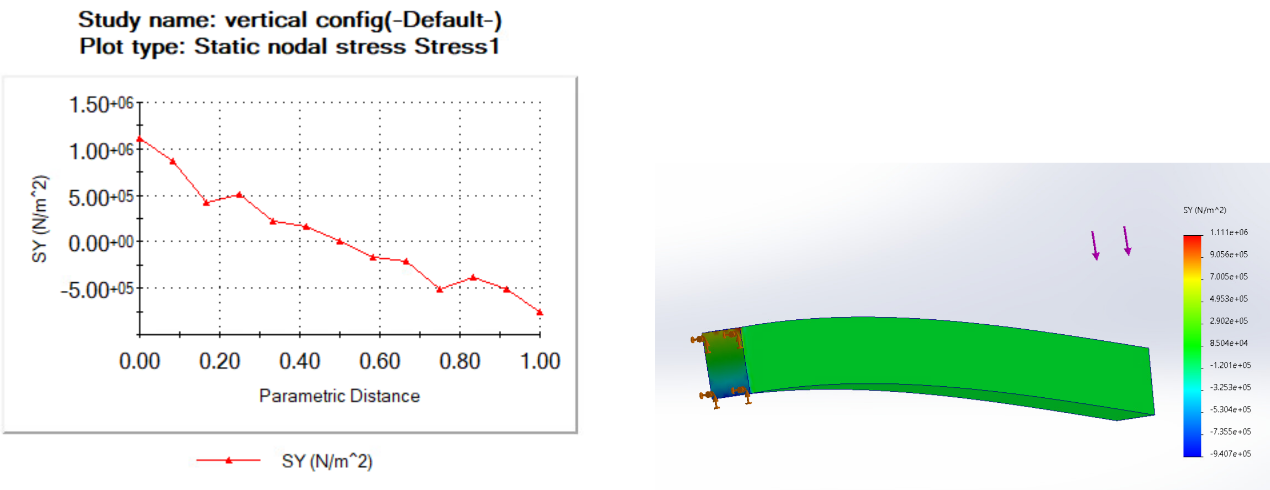
- y-displacement: -0.576 mm
- normal stress: 1.111 Mpa



(a) Vertical Displacement Chart

(b) Simulation in SOLIDWORKS FEA

Figure 21: Vertical Displacement Measurements



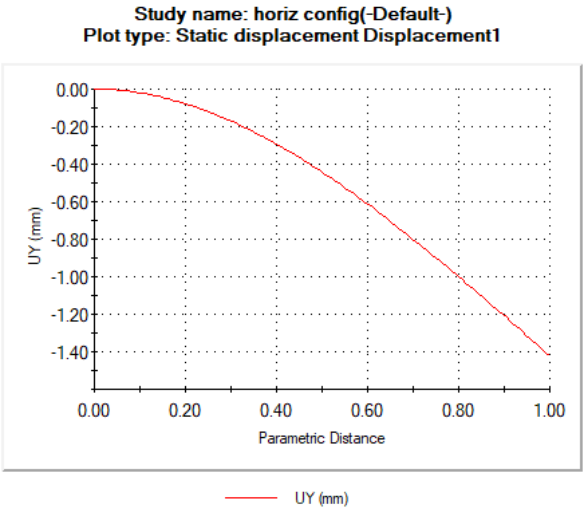
(a) Vertical Stress

(b) Simulation in SOLIDWORKS FEA

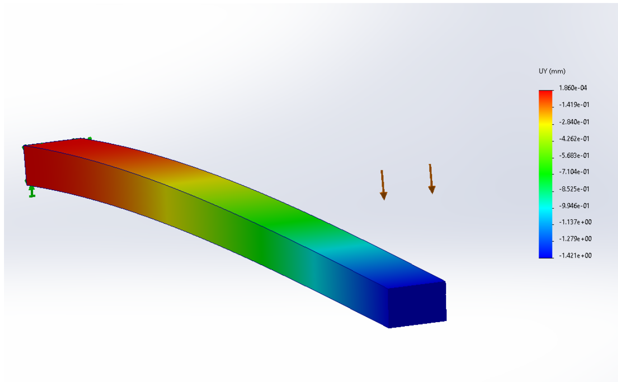
Figure 22: Vertical Stress Measurements

Horizontal configuration:

- y-displacement: -1.421 mm
- normal stress: 2.063 Mpa

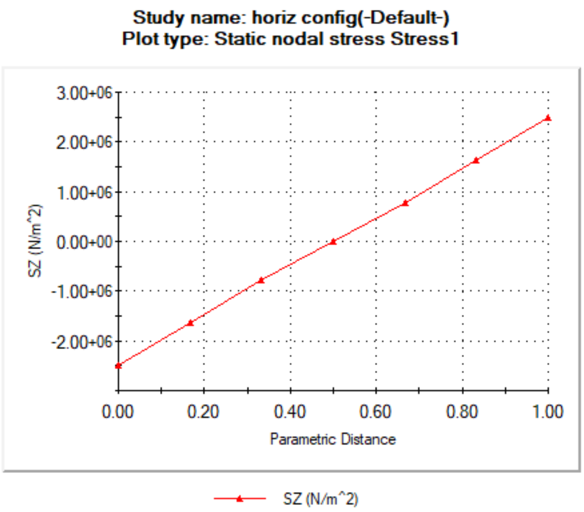


(a) Horizontal Displacement Chart

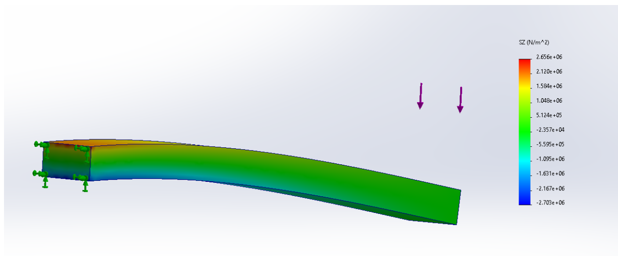


(b) Simulation in SOLIDWORKS FEA

Figure 23: Horizontal Displacement Measurements



(a) Horizontal Stress



(b) Simulation in SOLIDWORKS FEA

Figure 24: Horizontal Stress Measurements

Comparison

*See attached Excel File

Part 3: Visualize the strain field for the acrylic beam

(a) Polar Acrylic 1 (b) Polar Acrylic 2 (c) Zoomed in view

Figure 25: Polariscope visualization of the acrylic beam strain field.

As stress is applied to the beam, the area around which the beam is mounted experiences variable amounts of stress. The stress concentration appears to be closest to the side of the acrylic that is mounted/pressed against the base of the supporting mount. This is because the primary shear force occurs at this point since it is experiencing a moment.

Conclusion

This project demonstrated how different support conditions affect the deflection of a material by comparing the behavior of steel and aluminum under simply supported and cantilevered support conditions. We also observed how moment of inertia affects beam deflection by measuring an acrylic beam in two different orientations. As expected, aluminum consistently deflected more than steel under every loading condition due to its significantly lower Modulus of Elasticity.

Our theoretical calculations and FEA results generally agreed with each other. The simply supported experimental deflections were close to the expected deflection. However, the cantilevered experimental deflections showed a consistent and substantial difference (about a 50% difference for both materials) compared to both the FEA and the theoretical calculations. Since the error was proportional for both steel and aluminum, it indicates that a similar error affected the measurements for both of these loading conditions. It is likely that when we performed the experimental calculations for the deflection of the cantilevered beam, either the measuring tool was not properly calibrated, or we placed the load too quickly onto the measuring tool, causing both the steel and the aluminum to deform more than expected.

Regarding support conditions, although we used different locations for the load for each of the setups, we can still conclude that a simply supported beam provides greater resistance to deflection than a cantilevered beam because it has two support forces at each end of the beam rather than the cantilevers singular support force and moment. Thus, a simply supported beam will displace less. Finally, the acrylic beam tests confirmed that a greater moment of inertia results in a decrease in beam deflection, demonstrating the importance of beam orientation.