FEA Lab

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Deliverable 1: Vertical Displacement Plot

The maximum vertical displacement of the cantilever beam under the applied load is shown in the displacement plot. The color scale indicates the magnitude of the displacement, with the maximum value of 0.0189in occurring at the free end.

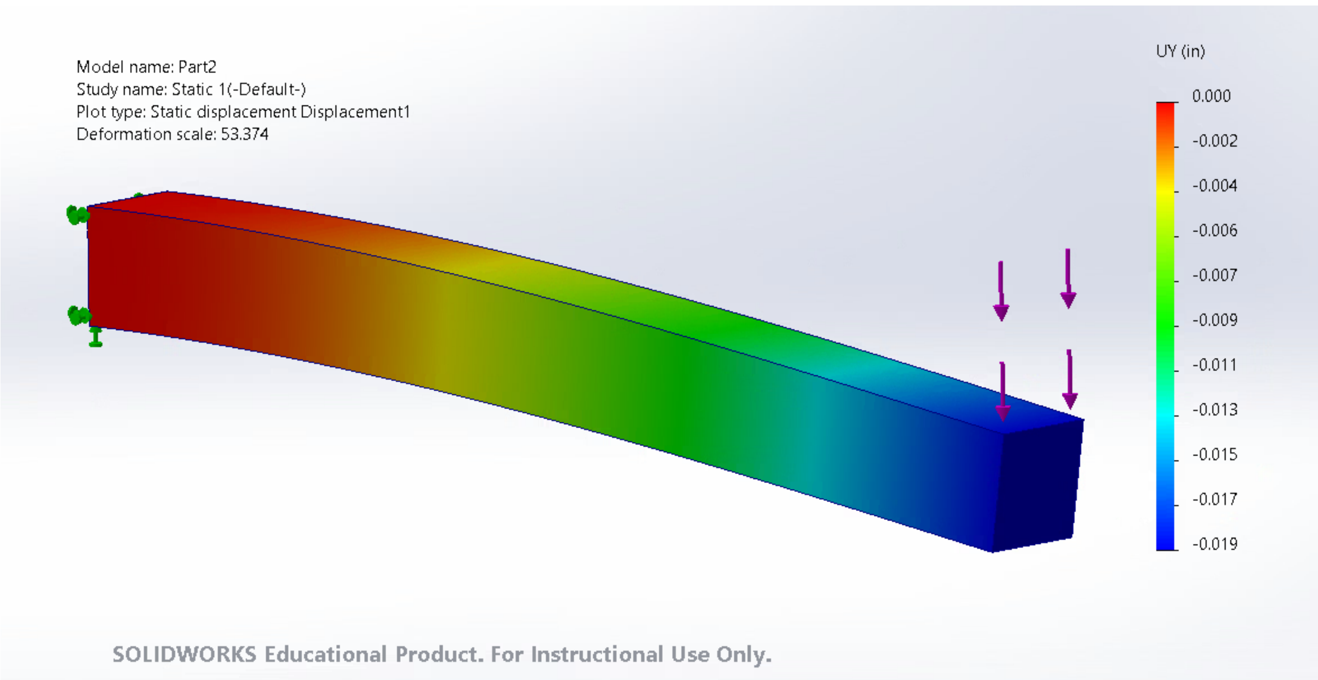

Figure 1: Vertical Displacement Plot (inches) of the Cantilever Beam

Deliverable 2: Deflection Plot along the Beam Edge

The deflection along the entire beam edge was probed and plotted as a function of the parametric distance from the fixed end. This plot illustrates the deflection curve, with zero slope and deflection at the fixed support ($x = 0$). The maximum vertical deflection from the FEA results is:

$$\delta_{\text{FEA, max}} = 0.0189 \text{ in}$$

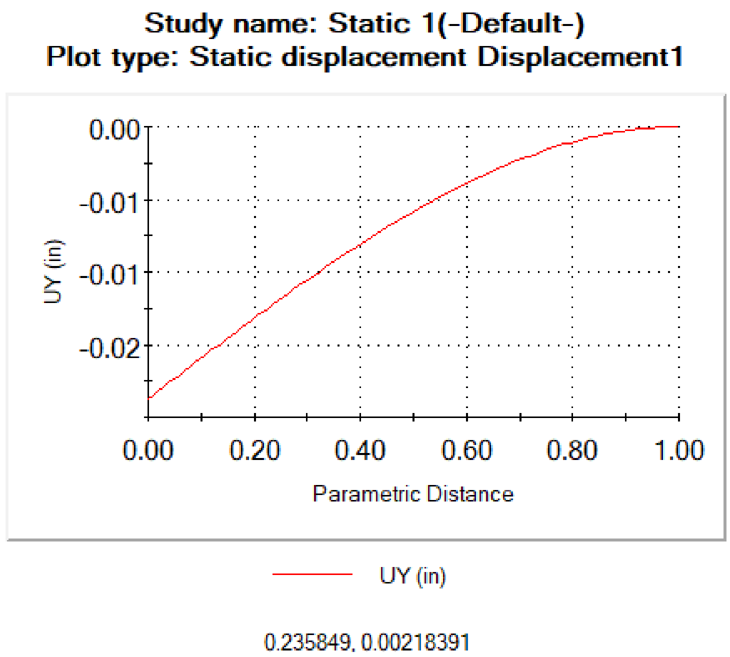

Figure 2: Vertical Deflection (in) along the Beam Edge

Deliverable 3: Maximum Deflection Verification

The maximum deflection from the FEA is verified against the analytical solution for a cantilever beam with a point load (P) at the free end: Given: $P = 75$ lbs, $L = 10$ in, $E = 16e6$ PSI. $I = \frac{d^3 b}{12} = 0.08333$ in^4

$$\delta_{\max} = \frac{PL^3}{3EI} = \frac{(75)(10)^3}{3(16e6)(0.08333)} = 0.0188\,\text{in}$$

Comparison:

- FEA Maximum Deflection ($\delta_{\text{FEA, max}}$): 0.0189 in
- Analytical Maximum Deflection ($\delta_{\text{analytical, max}}$): 0.0188 in
- Percentage Difference: 0.5% So they are basically the same!

The FEA result agrees well with the beam theory, confirming the accuracy of the model's deflection results.

Deliverable 4: Von Mises Stress Plot

The maximum von Mises stress is expected to occur at the fixed support, specifically at the top or bottom edges, where the bending moment is maximum (as seen in the simulation).

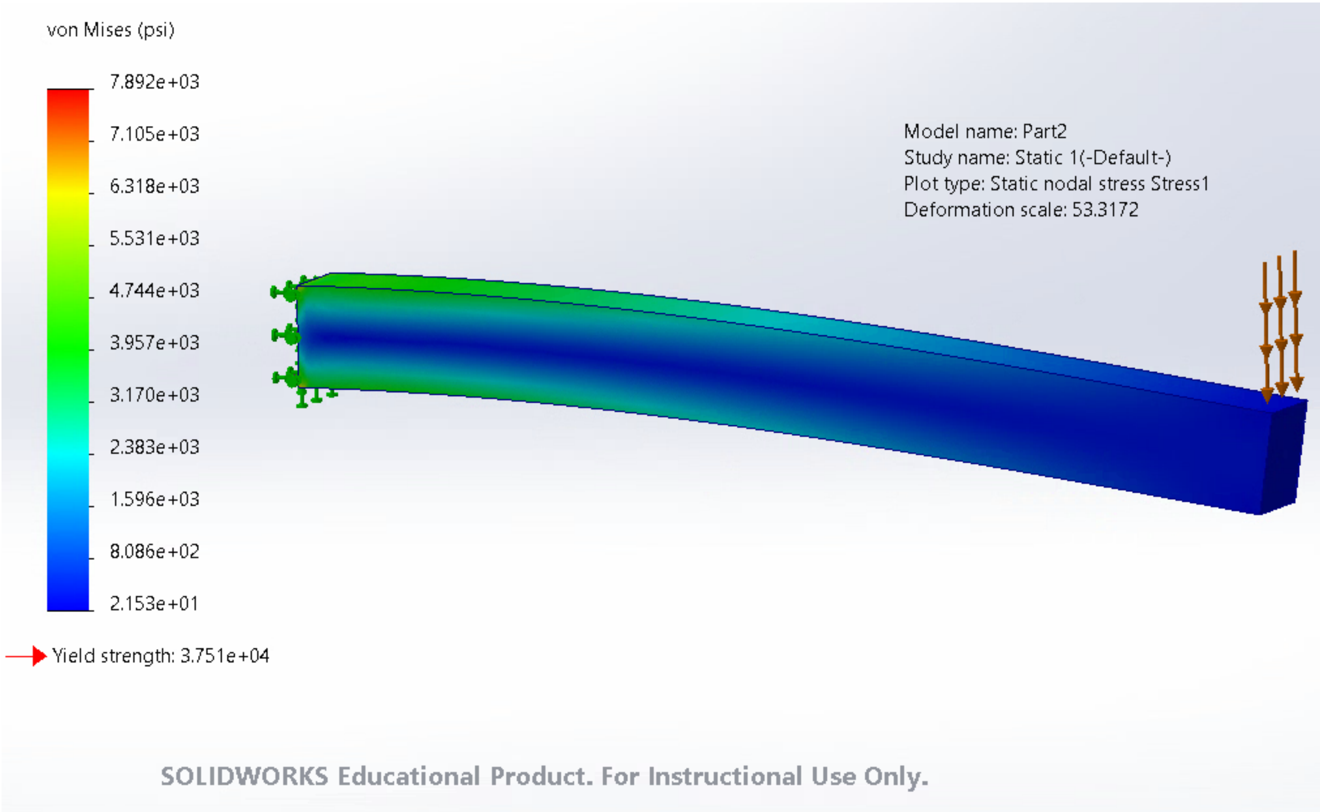

Figure 3: Von Mises Stress Plot (PSI) of the Cantilever Beam

Deliverable 5: Stress Variation Plot along the Top Edge

The von Mises stress along the top edge of the beam is plotted as a function of distance from the fixed end ($x = 0$). The maximum von Mises stress from the FEA is:

$$\sigma_{\text{Von Mises, max}} = 4350\,\text{PSI}$$

(read from the peak before the simulation becomes inaccurate).

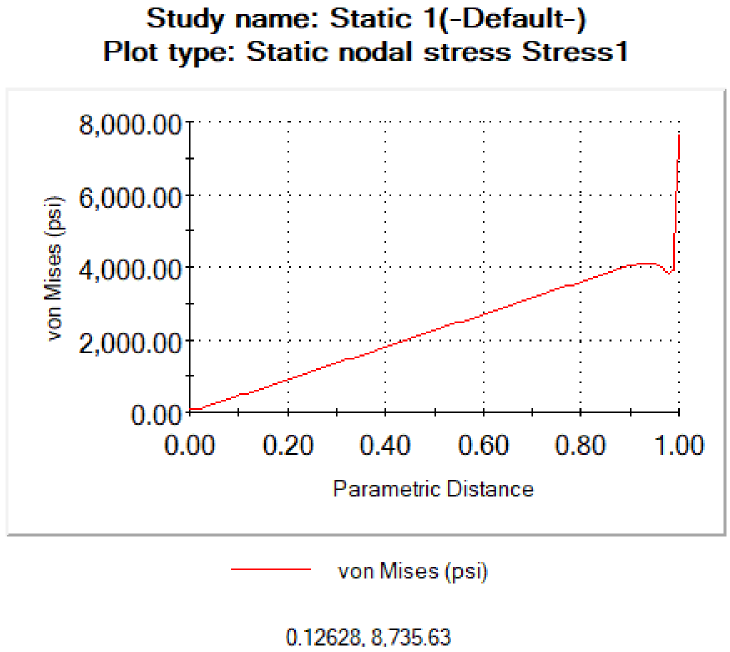

Figure 4: Von Mises Stress (PSI) along the Top Edge (Original BC)

Deliverable 6: Maximum Stress Verification

The maximum stress at the wall from the FEA is verified against the analytical bending stress formula. The maximum bending moment is $M_{\max} = PL$. Given: $M_1 = 75(10) = 750$ lb $\cdot$ in, $c = 0.5$ in, $I = \frac{b^3 d}{12} = 0.08333$ in^4

$$\sigma_{\max} = \frac{M_{\max} c}{I} = \frac{(750)(0.5)}{0.0833} = 4500\,\text{PSI}$$

Comparison:

- FEA Maximum Von Mises Stress ($\sigma_{\text{FEA, max}}$): 4150 PSI
- Analytical Maximum Bending Stress ($\sigma_{\text{analytical, max}}$): 4500 PSI
- Percentage Difference: 8.43%

The FEA result is somewhat close to the analytical stress. Note that the FEA often shows a higher stress at the boundary due to the stress concentration at the sharp corner of the fixed support, which is not represented to a high level of accuracy with the fixed boundary condition.

Deliverable 7: Comparison of Stresses with Modified Boundary Conditions

The boundary condition was changed from a rigid fixed support to the modified boundary condition (intended to better represent a physical joint). The stress components (σ_x, σ_y, σ_z) are plotted and compared to the Von Mises stress (σ_{vM}).

Stress Plots with Modified Boundary Condition

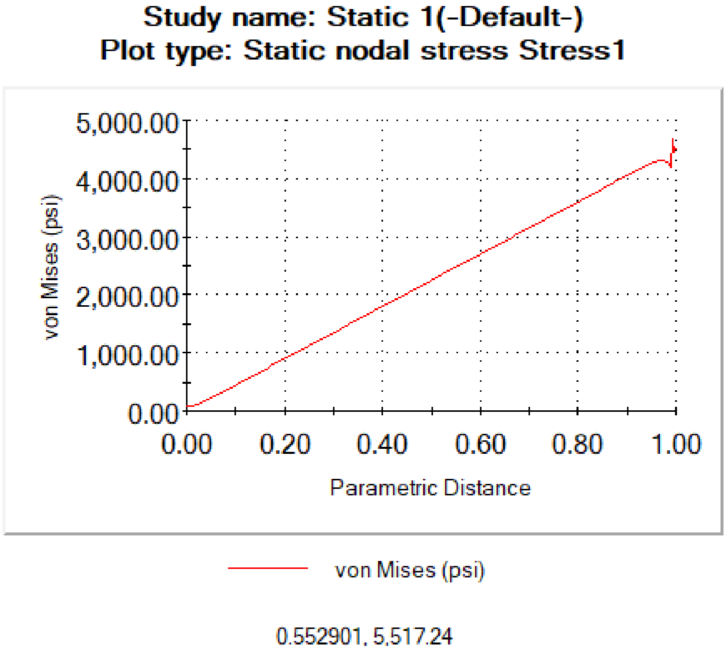

Figure 5: Von Mises Stress (PSI) along the Top Edge with Modified BC

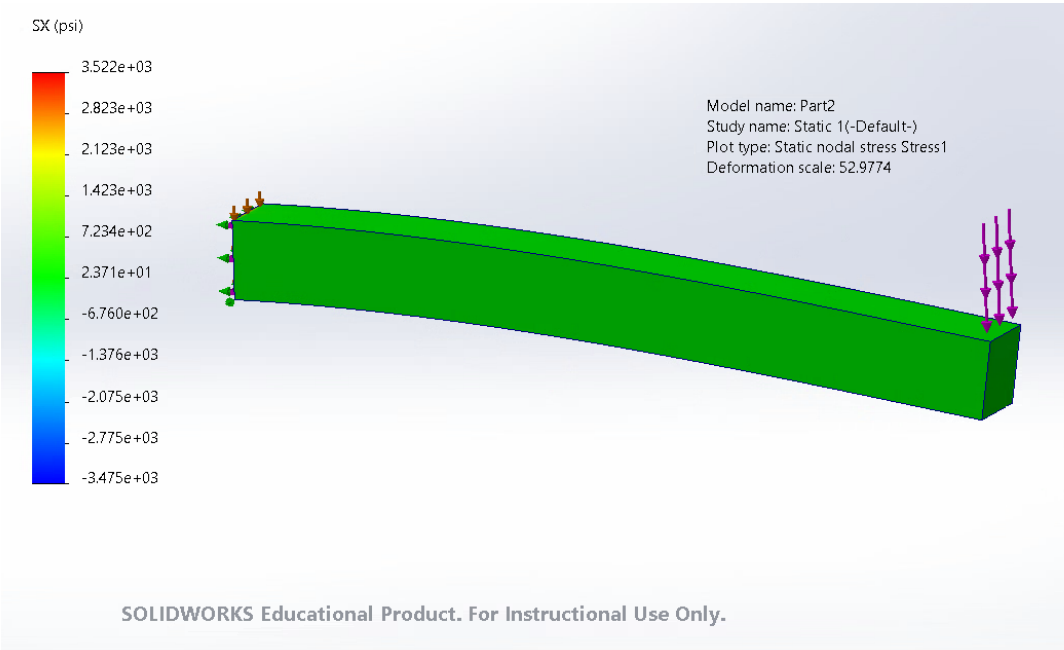

Figure 6: Normal Stress in X-direction (σ_x) (Modified BC)

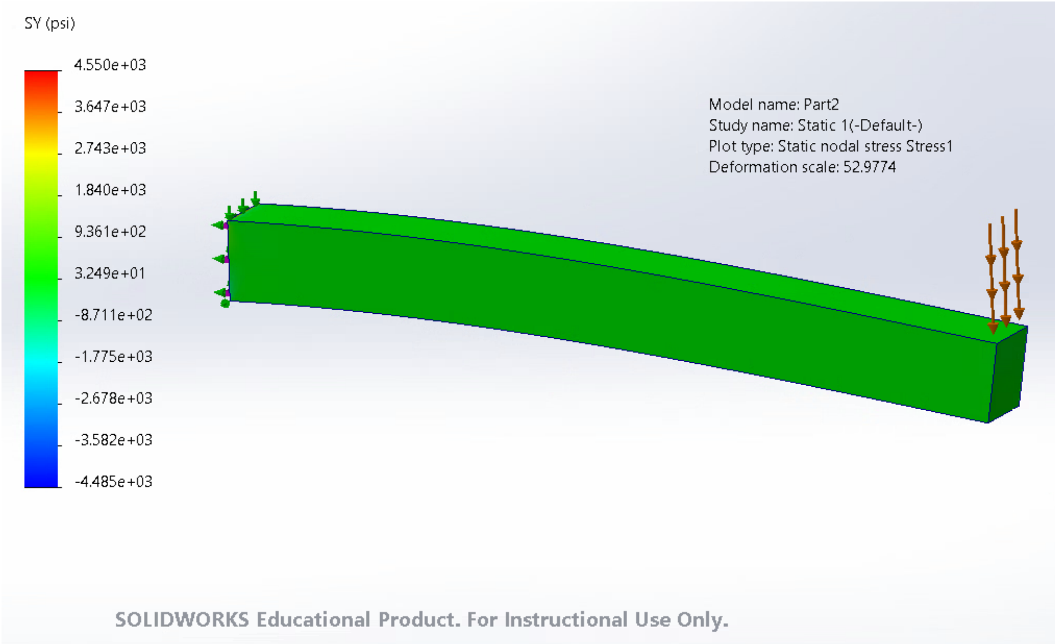

Figure 7: Normal Stress in Y-direction (σ_y) (Modified BC)

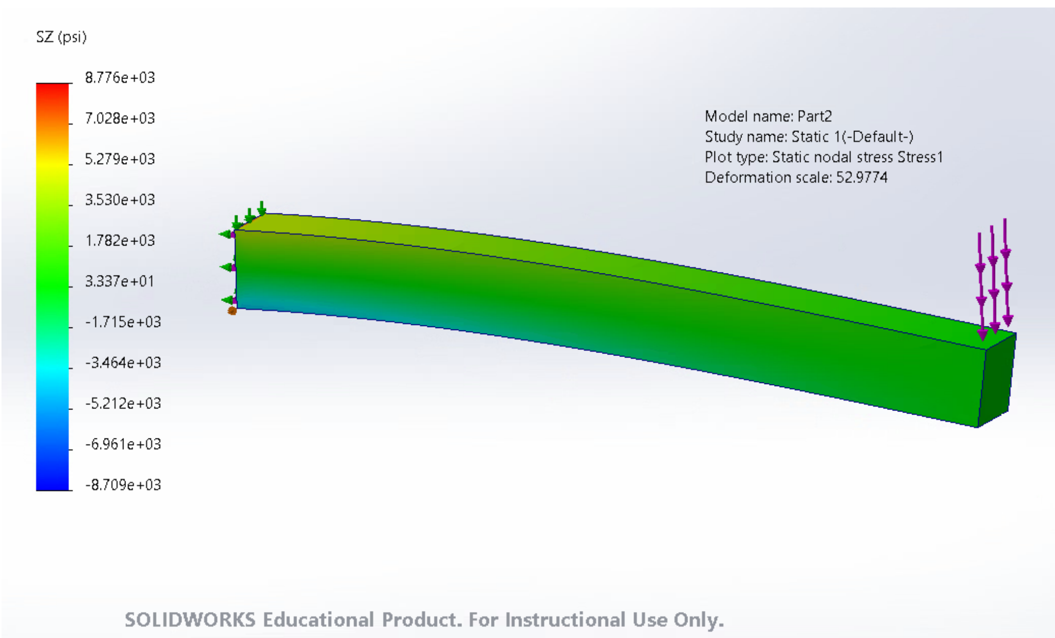

Figure 8: Normal Stress in Z-direction (σ_z) (Modified BC)

Commentary on Results

The σ_z stress is the dominant stress component. For this bending case, the von Mises stress (σ_{vM}) seems to be approximately equal to the magnitude of the dominant normal stress, $\sigma_{\text{vM}} \approx |\sigma_z|$. The transverse normal stresses σ_y and σ_x are relatively small. The modified boundary condition seems to lead to a smoother stress profile near the support and a reduced, more realistic stress concentration compared to the rigid Fixed boundary condition.

Deliverable 8: Largest Load to Avoid Yield

The largest safe load (P_{safe}) was calculated by scaling the applied load (P_{applied}) by the ratio of the yield strength to the maximum von Mises stress observed:

$$P_{\text{safe}} = P_{\text{applied}} \times \left(\frac{\sigma_{\text{yield}}}{\sigma_{\text{vM, max, applied}}} \right)$$

From the analysis (using the most accurate values)s:

- Applied Load (P_{applied}): 75 lbs
- Maximum Von Mises Stress from FEA ($\sigma_{\text{vM, max, applied}}$): 4500 PSI
- Material Yield Strength (σ_{yield}): 26000 PSI

$$P_{\text{safe}} = 75\,\text{lbs} \times \left(\frac{26000}{4500} \right) = 443\,\text{lbs}$$

The model can safely support a maximum load of 443 lbs before the material begins to yield. Note that the yield strength (PSI) of the material in SOLIDWORKS appears to differ from the given material yield strength, which seems to emphasize that the new boundary still has some inaccuracies.

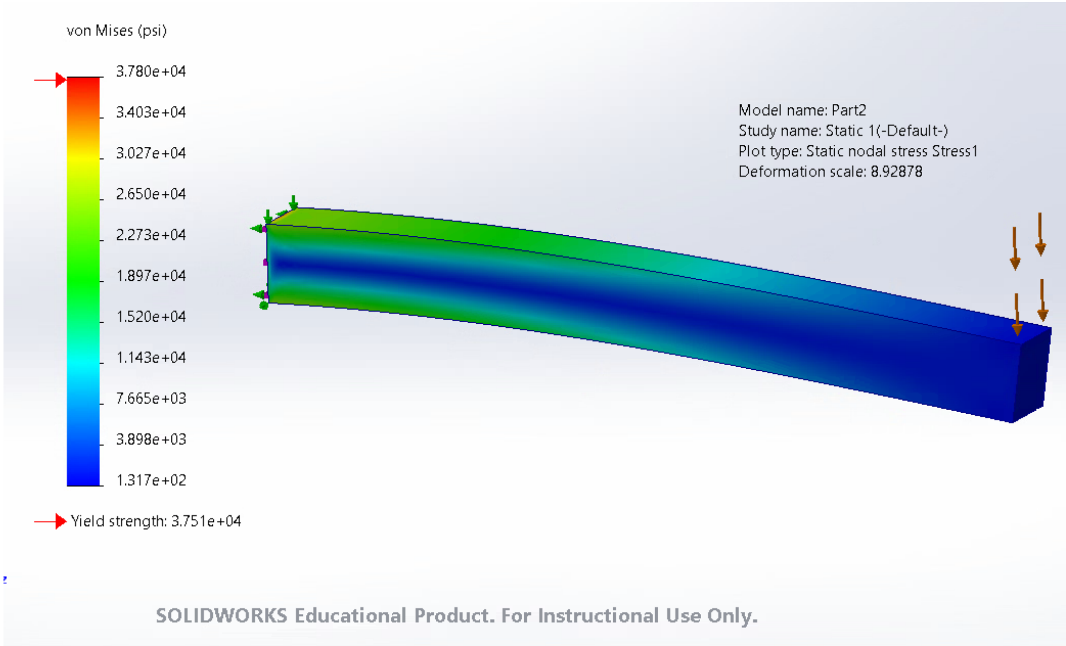

Figure 9: Stress plot showing the maximum Von Mises stress just after yield